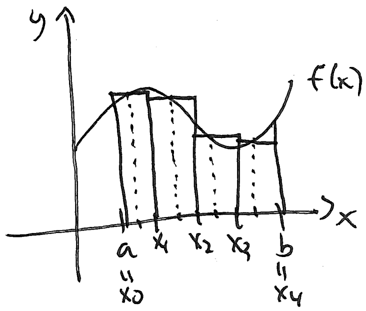
Worksheet 5 Solutions

1 a) $M_n = \Delta x \cdot [f(\bar{x}_1) + f(\bar{x}_2) + \dots + f(\bar{x}_n)]$, where $\Delta x = \frac{b-a}{n}$, $x_i = a + i\Delta x$, $\bar{x}_i = \frac{x_i + x_{i-1}}{2}$



b) $|E_{M_n}| \le \frac{K(b-a)^3}{24n^2}$. Want $\left|\frac{d^2}{dx^2}(\sin(x))\right| = |-\sin(x)| = |\sin(x)| \le K$,

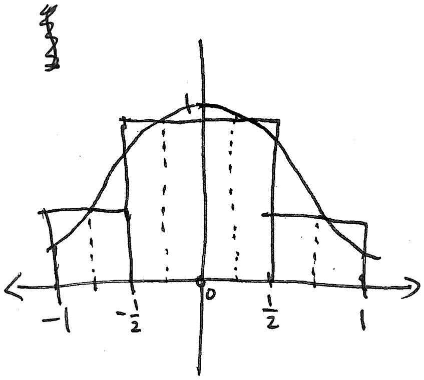
so choose $K = 1$,

(since $-1 \le \sin(x) \le 1$)

$\frac{1 \cdot (1-0)^3}{24n^2} < 10^{-7} \Rightarrow n > 645.497$

$\boxed{n = 646}$

2)

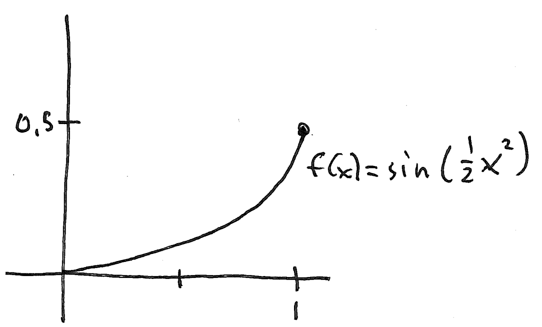


$\int_{-1}^{1} e^{-x^2}\,dx \approx M_4 = 0.5\left(e^{-(-\frac{3}{4})^2} + e^{-(-\frac{1}{4})^2} + e^{-(\frac{1}{4})^2} + e^{-(\frac{3}{4})^2}\right)$

≈ 1.509196

This is an overestimate, because e^{-x^2} is mostly concave down on $[-1,1]$ and the midpoint rule overestimates concave down functions.

3)



a) L_2 and M_2 underestimate, R_2 and T_2 overestimate.

b) (for this function only!!)

$L_n \le M_n \le I \le T_n \le R_n$

Side Note:

- f is increasing $\Rightarrow$ L_n underestimates, R_n overestimates
- f is decreasing $\Rightarrow$ R_n underestimates, L_n overestimates
- f is concave up $\Rightarrow$ M_n underestimates, T_n overestimates
- f is concave down $\Rightarrow$ T_n underestimates, M_n overestimates

c) $L_5 = 0.2\left(\sin(0) + \sin(\frac{1}{2}(0.2)^2) + \sin(\frac{1}{2}(0.4)^2) + \sin(\frac{1}{2}(0.6)^2) + \sin(\frac{1}{2}(0.8)^2)\right)$

≈ 0.118702

$R_5 = 0.2\left(\sin(\frac{1}{2}(0.2)^2) + \sin(\frac{1}{2}(0.4)^2) + \sin(\frac{1}{2}(0.6)^2) + \sin(\frac{1}{2}(0.8)^2) + \sin(\frac{1}{2}(1)^2)\right)$

≈ 0.214587

$M_5 = 0.2\left(\sin(\frac{1}{2}(0.1)^2) + \sin(\frac{1}{2}(0.3)^2) + \sin(\frac{1}{2}(0.5)^2) + \sin(\frac{1}{2}(0.7)^2) + \sin(\frac{1}{2}(0.9)^2)\right)$

≈ 0.162247

$T_5 = \frac{1}{2}(L_5 + R_5)$

≈ 0.166644

- $\boxed{M_5}$

4] $M_3 = 2\left(v(1)+v(3)+v(5)\right) = 2(1.34+1.9+3.2) = \boxed{12.88\text{ m}}$

$T_6 = \frac{1}{2}\cdot 1\cdot\left(v(0)+2v(1)+2v(2)+2v(3)+2v(4)+2v(5)+v(6)\right)$

$= \frac{1}{2}(.75+2.68+3+3.8+5+6.4+3)$

$= \boxed{12.315\text{ m}}$

5] $|E_{S_n}| \leq \frac{C\cdot(2-1)^5}{180\,n^4}$, want $|f^{(4)}(x)| = \left|\frac{24}{x^5}\right| \leq C$ on $[1,2]$: $C=24$

$\frac{24}{180n^4} < 10^{-3}$

$n^4 > \frac{24{,}000}{180}$

$n > 3.39$

$\underline{n=4}$

$S_4 = \frac{\Delta x}{3}\left(f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+f(x_4)\right)$

$= \frac{0.25}{3}\left(\frac{1}{1}+4\cdot\frac{1}{1.25}+2\cdot\frac{1}{1.5}+4\cdot\frac{1}{1.75}+\frac{1}{2}\right)$

≈ 0.69325397

$\int_1^2 \frac{1}{x}\,dx = \ln 2 \approx 0.693147$

error $= \ln 2 - S_4 \approx 0.000107 < 10^{-3}$

$\approx 10^{-4}$